Lesson Plan 3: Stealthy Submarine: How Close Will The Enemy Get?
(Bearing, Range and Angle on the Bow)

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2013 Submarine Force Library and Museum Association STEM Teacher Fellow

Instructional Goal

Algebra and Geometry teachers who are looking for an application activity, which uses trigonometric equations for sine and cosine for finding the length of a side of a triangle or the distance between two points.

Common Core State Mathematics Standards:

- HSG-SRT.D.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles.

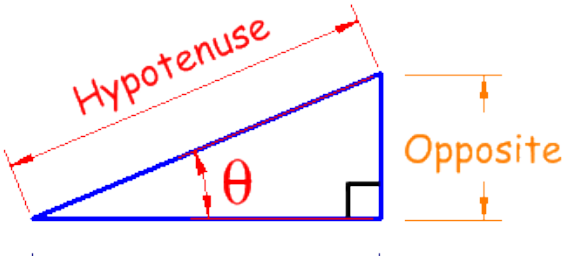
Students will learn about the fire control systems historically used on submarines, and specifically the capability of a piece of equipment called the "Position Keeper".

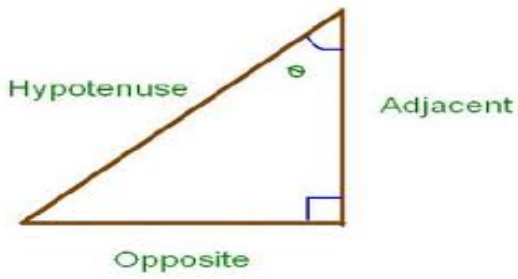
Students will learn how to estimate how close an enemy vessel may get, based on information they can obtain from the Position Keeper, and will learn how a submarine tracks a threat target based on their current location, their bearing, and the threat vessel's current direction, distance away, and the angle on the bow.

Background Knowledge

In this lesson students will use the sine and the cosine to determine the hypotenuse and missing leg of a right triangle. Students will see how applying trigonometry was one of the skills that helped sailors to stay safe while on a submarine. Students will discuss the different relationships between the side lengths and the angles of a right triangle. Teachers are free to expand or shorten the lesson as needed.

Students should understand that the side called "opposite" and the side called "adjacent" will change depending on which angle θ of the right triangle is used for calculation of sine or cosine.

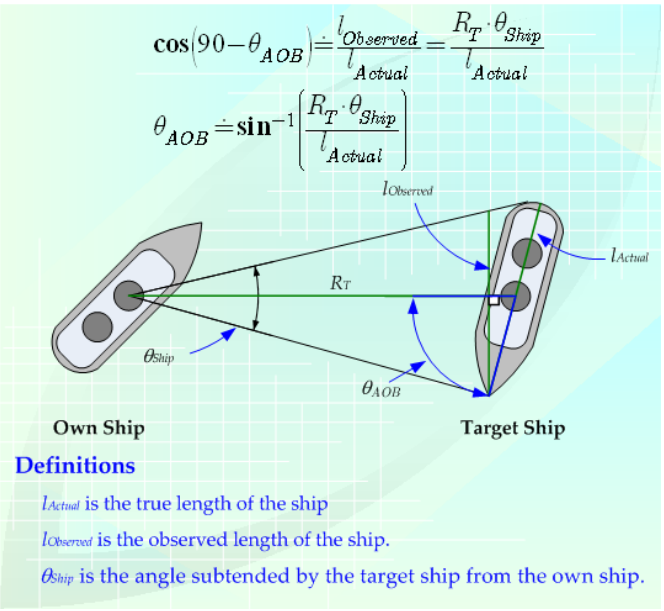




Navy Background

See http://www.ussnautilus.org/virtualTour/attack.shtml and the periscope room 360 degree panorama at http://www.ussnautilus.org/virtualTour/nautilus/_flash/USS%20Nautilus_nautilus.html for the Submarine Force Museum and Historic Ship Nautilus attack center.

For details of the torpedo data computer see:
http://maritime.org/tech/tdc.htm and
http://en.wikipedia.org/wiki/Torpedo_Data_Computer
and for Fire Control basics see http://www.hnsa.org/doc/attack/ chapter one, and PLATE I figure 1 (below).



Target Ship

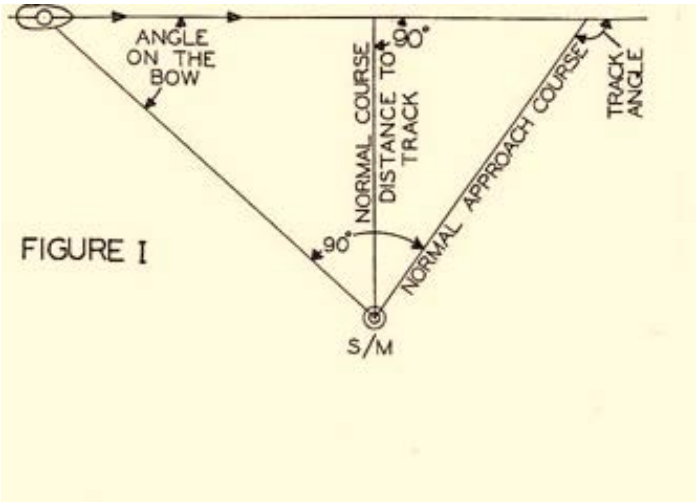


FIGURE I

Lesson Activities Included in the Application Activities/Student Worksheet

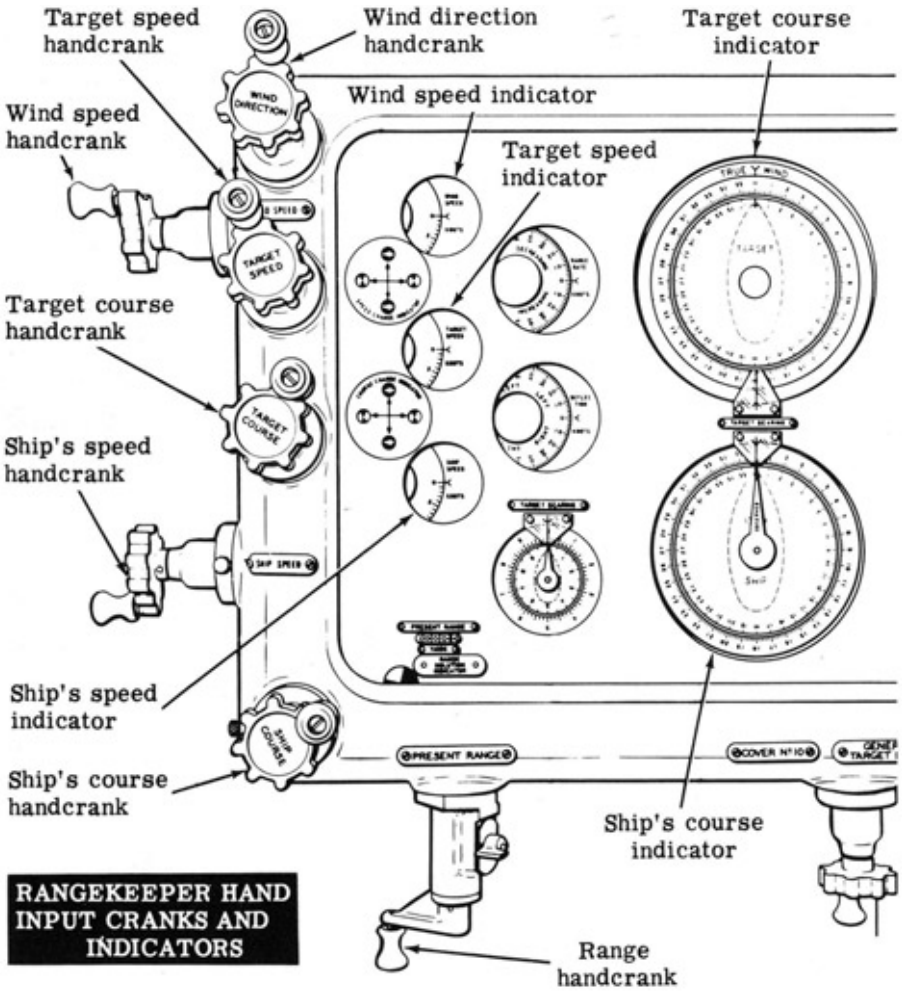
1. View a video on submarines at http://www.youtube.com/watch?v=zTohYRUt_2c . Modern computers do the calculations for you, but it is still necessary to understand the mathematics behind the calculations. In the earlier days before modern computers, sailors did the mathematical calculations by memory or by hand.

2. Discuss the information the position keeper provides. Discuss what "Bearing" and "Angle on the Bow" mean. See θ_{AOB} in the above diagram. AOB means "Angle on the Bow".

3. Use information from an annotated torpedo data computer to determine how far away the enemy vessel is. From the information, create a diagram that represents your submarine, the enemy vessel, the bearing and range of the enemy, and BOTH vessels' courses.

4. Discuss what the figure represents. What does each side of the triangle represent? What do different points along the hypotenuse represent? Discuss what changing the angle on the bow does to the hypotenuse and 2nd leg of the right triangle. How does this relate to the submarine and enemy vessel?

Navy Fire Control Diagram (old TDC)—a mechanical computer with electrical inputs/outputs



Application Activities

View a video on Submarine Operations http://www.youtube.com/watch?v=zTohYRUt_2c and another video on submarine Fire Control (http://www.youtube.com/watch?v=-JP-ytvCRuk) Modern computers do the calculations for you, but it is still necessary to understand the mathematics behind the calculations. In the earlier days before modern computers, sailors did the mathematical calculations by memory or by hand.

Students, use the attached worksheet **"How Close Will The Enemy Get?** to answer the following questions.

2. Discuss the information the position keeper provides. Discuss what "Bearing" and "Angle on the Bow" mean.

3. Gather the information from the diagram to determine how far away the threat vessel is. Create a diagram that represents your submarine, the threat vessel, the bearing and range of the enemy, and BOTH vessels' courses.

a) On your graph paper, based on how far away the threat vessel is, create a scale for the x and y axis, and label each axis.

b) Now, plot a point to represent your submarine. (Pt 1)

c) Plot a second point to represent the threat vessel. (Pt 2) Be sure to plot the 2 points on the same x value so that they are lined up vertically.

d) Draw a line that connects the 2 points. This is one leg of your right triangle. Label the length of this leg "RANGE". This is how far away the threat vessel currently is.

e) Draw a 90˚ angle from Pt 1.

f) From Pt. 2, draw a line that creates a 30 degree angle. This is called a "Starboard (right) Angle On The Bow.

f) Using these 2 angles draw the hypotenuse and the second leg of the triangle.

4. Discuss what the figure represents. What does each side of the triangle represent? What do different points along the hypotenuse represent?

5. You are aboard a submarine that uses a position keeper. You have just read the position keeper, and have recorded that there is an enemy vessel that is 6000 yards bearing 000 off your starboard bow. Your course is 270˚. The enemy vessel has a starboard 25 degree angle on the bow. Your ship control room loses power. You have to track the enemy ship manually and determine how close he will actually get to the submarine if he stays his course, and you do not change position. Use the sine and cosine functions on your graphing calculator to calculate the length of the sides of the right triangle. (You may have to make more than one calculation.).

6. When power is restored to the control room, you observe that although you have not moved, and your course is still 270˚, the enemy vessel is now 5000 yards away from you bearing 000, but his angle on the bow is now starboard 50˚. Draw a new representative right triangle, and determine the length of all the sides. How close will the enemy vessel get?

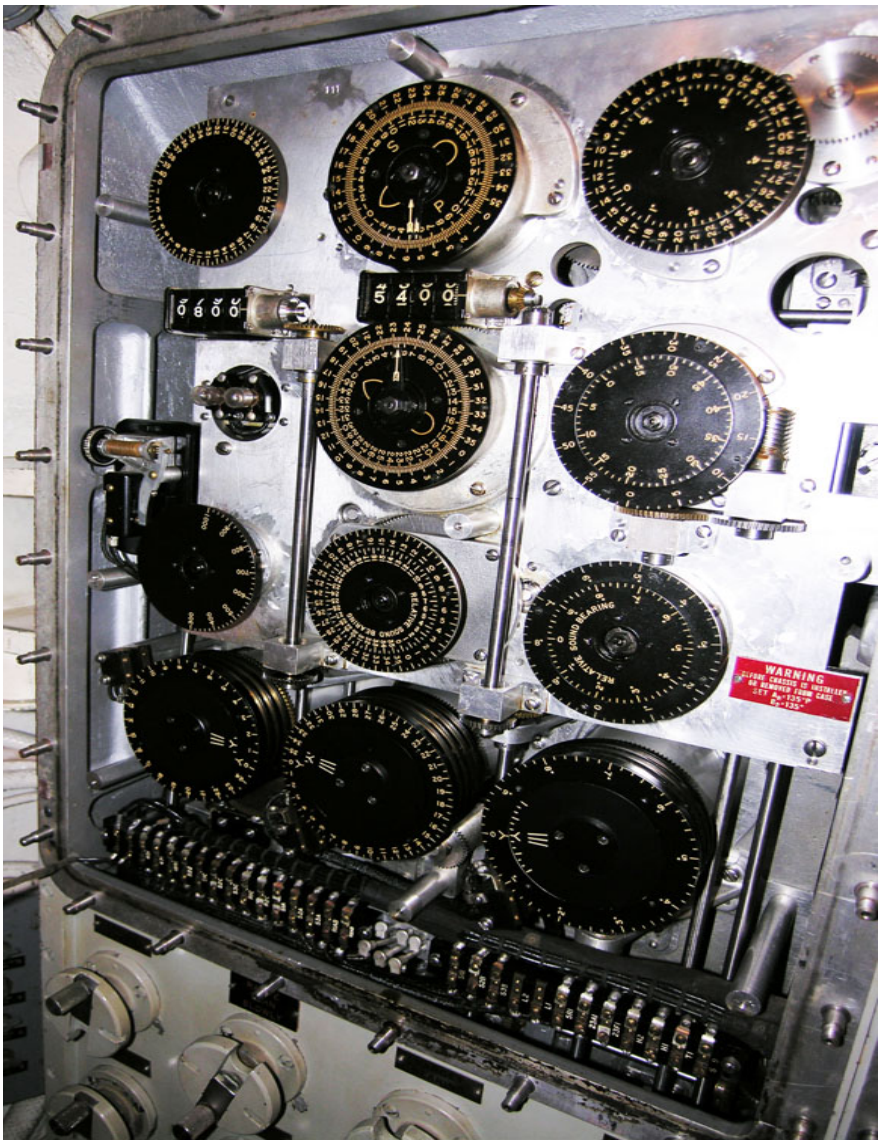
7. The enemy vessel has stopped moving. You are concerned that if they resume their course, that they will get too close to you. You decide to change your course to 240˚ and get under way. You travel for 10 minutes at 12 knots. The enemy vessel also has changed course, and after 10 minutes you read the position keeper and see that your course is 240˚, the enemy ship now has a starboard 40 degree angle on the bow, and they are 7500 yards away. If you hold your position, and the enemy vessel stays on course, how close to you will they get now?

8. If you want to keep them at least 4000 yards away, have you been successful?

9. (Extension Problem) What would the angle on the bow need to be to keep the enemy vessel 4000 yards away? What could you do to make this happen?

10. (Extension Problem) Looking at Figure I n the diagrams above, what name is given to the value: Target Range times Angle on the Bow = R_T x θ_{AOB} ? Why is distance to the target track important?

Student Worksheet: "How Close Will The Enemy Get?



Position Keeper Readings

Target Course: 150˚

Ship's Course: 270˚

Target Range: 7000 yards

Angle on the bow: 30˚

1. Gather the information from the diagram to determine how far away the threat vessel is. Create a diagram that represents your submarine, the threat vessel, the bearing and range of the enemy, and BOTH vessels' courses.

g) On your graph paper, based on how far away the threat vessel is, create a scale for the x and y axis, and label each axis.

h) Now, plot a point to represent your submarine. (Pt 1)

i) Plot a second point to represent the threat vessel. (Pt 2) Be sure to plot the 2 points on the same x value so that they are lined up vertically.

j) Draw a line that connects the 2 points. This is one leg of your right triangle. Label the length of this leg "RANGE". This is how far away the threat vessel currently is.

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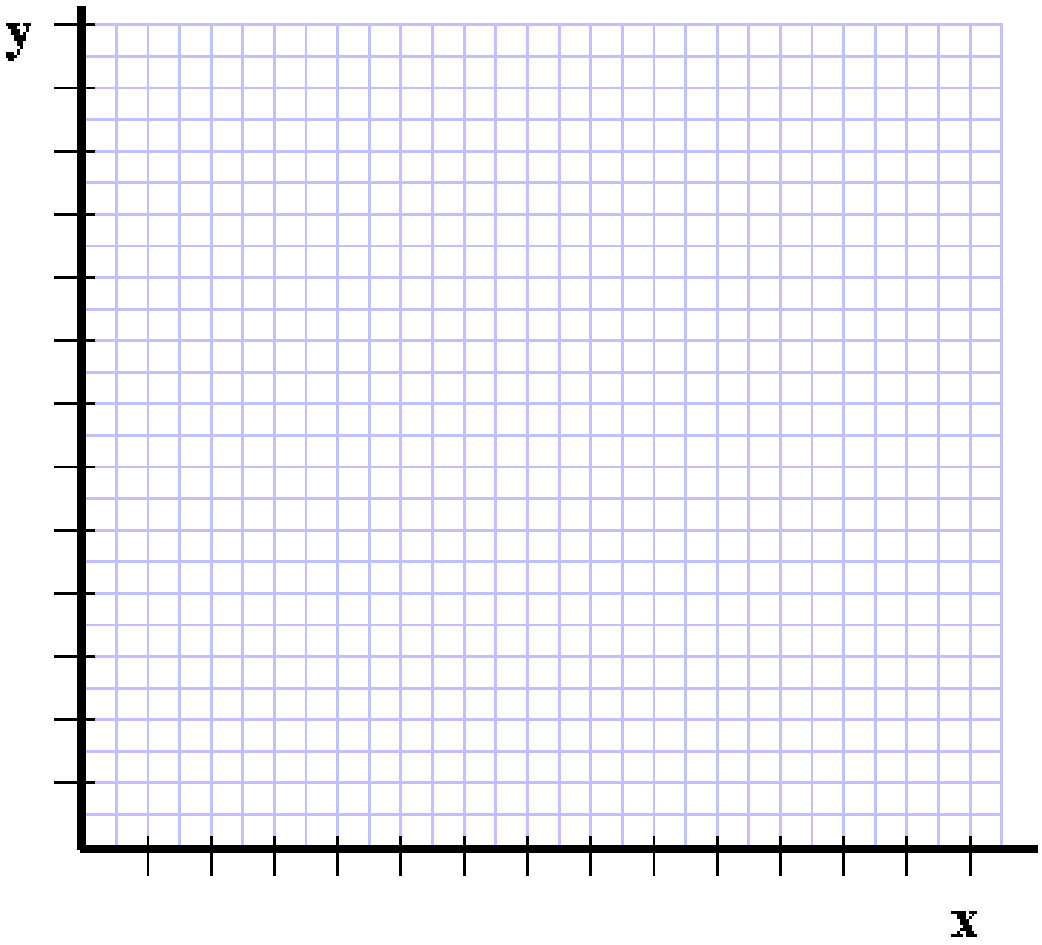
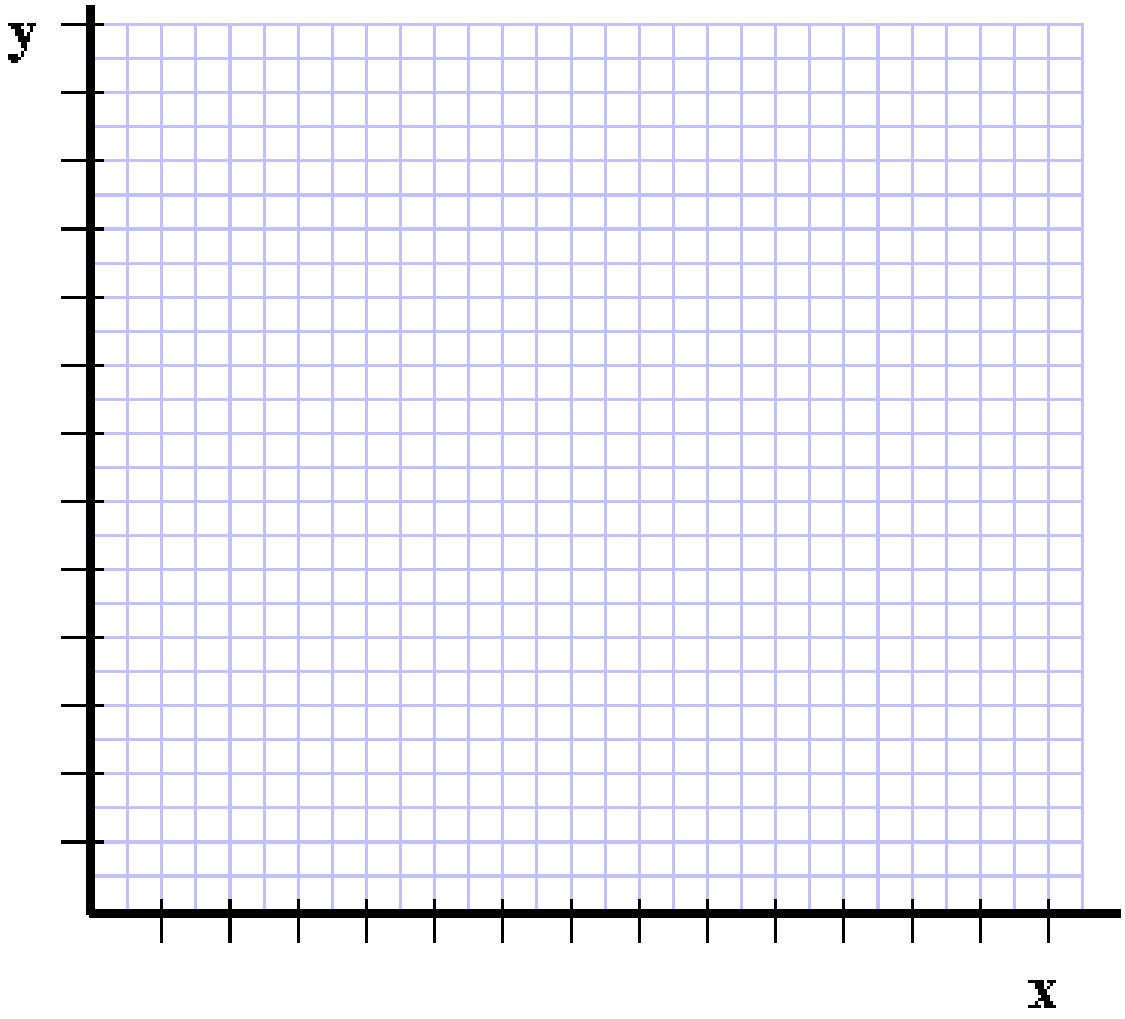
l) Using these 2 angles draw the hypotenuse and the second leg of the triangle.



2. Discuss what the figure represents. What does each side of the triangle represent? What do different points along the hypotenuse represent?

3. You are aboard a submarine that uses a position keeper. You have just read the position keeper, and have recorded that there is an enemy vessel that is 6000 yards off your starboard bow. Your course is 270˚. The enemy vessel has a starboard 35˚ angle on the bow. Your ship control room loses power. You have to track the enemy ship manually and determine how close he will actually get to the submarine if he stays his course, and you do not change position. Draw a diagram with the given information (Like the diagram in problem 1). Use the sine and cosine functions on your graphing calculator to calculate the length of the sides of the right triangle. (You may have to make more than one calculation.).



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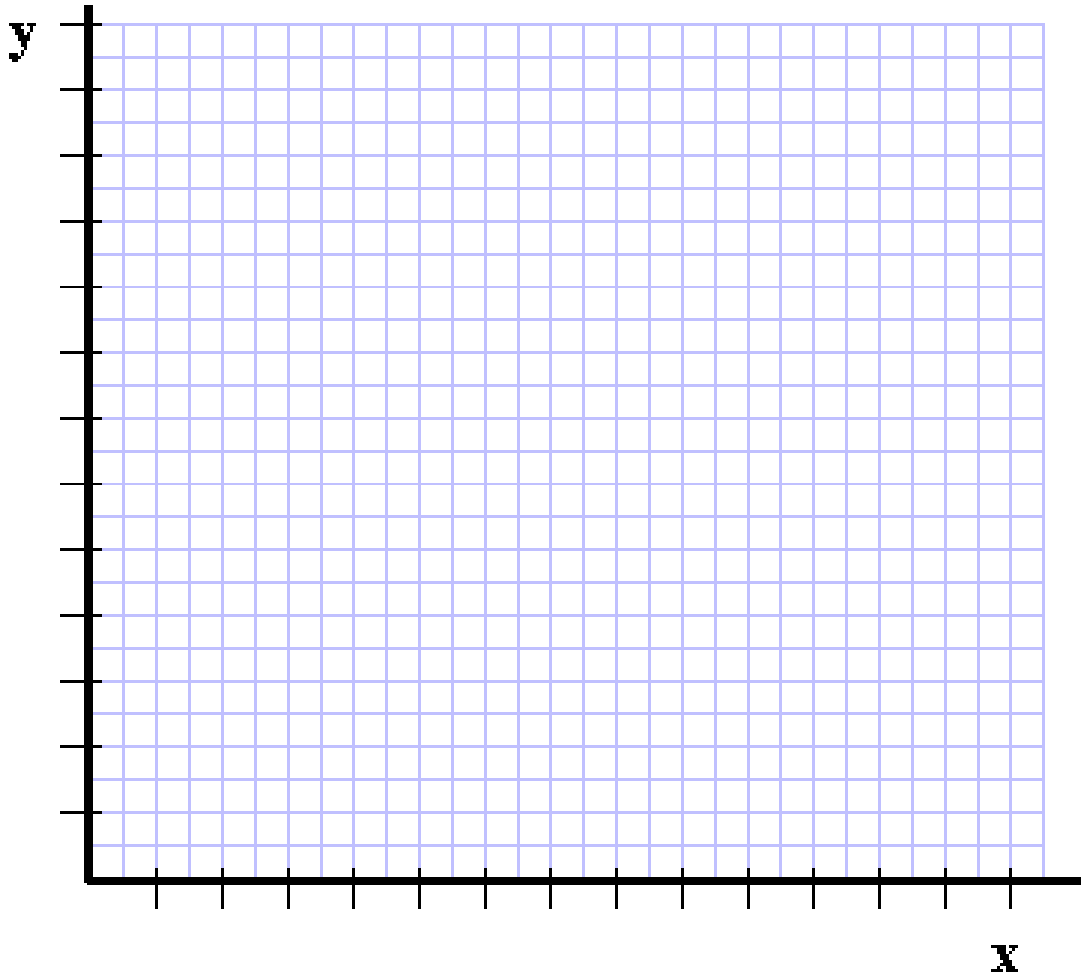

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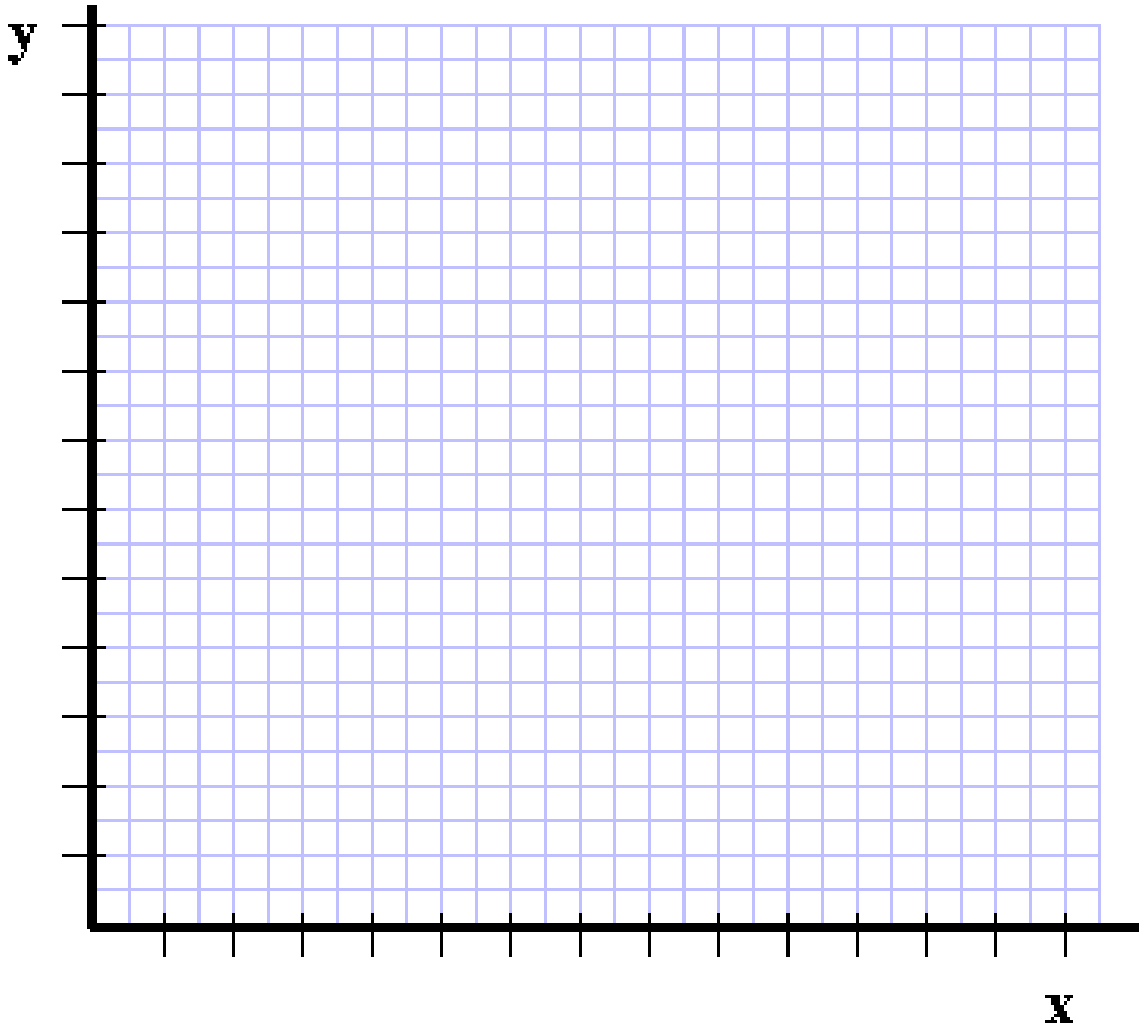

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